$^1D_2$ and $^3F_3$ dinucleon poles within the M-matrix formalism

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The $^1D_2$ dinucleon pole observed within the M-matrix formalism is subjected to a simple test to determine whether it can be of the bound-state type in the $N\Delta$ system in a conventional sense similar to the deuteron in the $^3S_1$ nucleon-nucleon scattering partial wave. This possibility is of interest because of the $^1D_2$'s highly inelastic nature (coupled mainly to the $N\Delta$ channel) and its close proximity to the $N\Delta$ branch point. Results for the observed $^3F_3$ pole are also reported.

The idea of dibaryons in the $^1D_2$ and the $^3F_3$ nucleon-nucleon partial waves has received a great deal of attention recently.[1–4] Papers[1,3,4] in their support show their existence through the presence of poles in T-matrix representations for the corresponding phase shifts. A commonly observed feature of these poles is their highly inelastic nature, being coupled mainly to the quasi-two-body $N\Delta$ channel and, in addition, lying close to the $N\Delta$ branch point in the complex energy plane. The authors of Ref. 1, not long ago, reported the observation of these poles in the M- and K-matrix energy-dependent fits to the new, precise phase shifts of Arndt *et al.*[5] They employed a coupled-channel formalism, expressing the reduced T matrix as

$$T^{-1}=A-i\rho\ , \tag{1}$$

where $A=M$ or K^{-1} and the elements of ρ are the phase-space factors

$$\rho_{11}=[(s-s_e)/(s-c_e)]^{l_e+1/2}\ , \tag{2a}$$

$$\rho_{22}=\frac{1}{(E-c_i)^{l_i+1/2}}\int_{M_T=M_N+M_\pi}^{\infty}\frac{(E-M_N-M)^{l_i+1/2}(M-M_T)^{3/2}dM}{(M+\alpha)^{l_i+2}[(M-M_0)^2+\Gamma^2/4]}\ , \tag{2b}$$

corresponding, respectively, to the two channels NN and $N\Delta$ considered in the analysis. The Mandelstam variable s equals the square of the center-of-mass energy E. l_e and l_i are the orbital angular momenta in elastic and inelastic channels, respectively. s_e is the elastic-threshold energy squared. The phenomenological parameters c_e and c_i have values less than 2 M_N. They control the behavior of phase-space factors, ensuring that they do not continue to rise sharply away from their respective thresholds. In addition, they contribute left-hand cuts to the amplitude. Similarly, the form factor $(M+\alpha)^{l_i+2}$ present in the denominator of the integrand in Eq. (2b) modifies, in the M dependence and in the same above sense, the threshold effects corresponding to the production of Δ (in conjunction with N) and its subsequent decay into π and N. In the complex E plane, it gives rise to cuts at $E=M_N-\alpha$ which lie on the same unphysical sheets as the $N\Delta$ and its complex-conjugate partner shown in Fig. 1. For values of α greater than $-M_N$, they lie to the left of the elastic threshold. The matrix elements M_{ij} or K_{ij} are represented by polynomials in the laboratory kinetic energy T_L. Table I gives typical sets of coefficients of such polynomials obtained in M-matrix fits to the $^1D_2$ and $^3F_3$ phases in Ref. 1. As reported therein, analytic continuation of such a fitted T matrix to different unphysical sheets from the physical region (see Fig. 1) reveals a pole in each one of the partial waves with significant influence on the real energy axis. Table II summarizes the results of such M-matrix fits to these partial

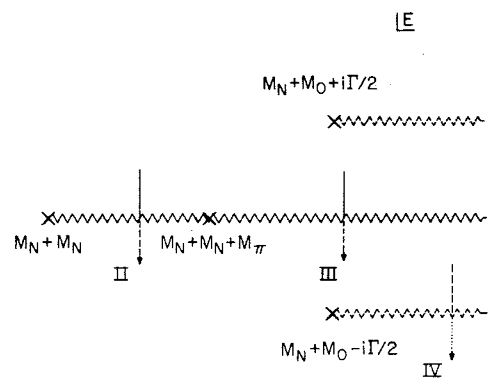

FIG. 1. Right-hand cut structure of the T matrix in the complex E plane.

TABLE I. Typical sets of parameters corresponding to the $^1D_2$ (best), $^1D_2$ (second best), and $^3F_3$ fits. Figures 3 and 4 show the poles given by these fits. The parameters B_m, C_m, and D_m are the coefficient of the term $(T_L)^{m-1}$ in the polynomial representations of the matrix elements M_{11}, M_{12}, and M_{22}, respectively. T_L is the laboratory kinetic energy in GeV. c_e, c_i, and α are the phase-space parameters [see Eq. (2) in the text].

	$^1D_2$ (best)	$^1D_2$ (second best)	$^3F_3$
B_1	−1.645	−2.404	3.848
B_2	0.028	1.861	−8.491
B_3	5.246	0.581	4.399
B_4	−3.120	0.0	0.0
C_1	−0.398	−0.594	−0.195
C_2	−0.019	0.151	0.439
C_3	0.763	−0.058	−0.343
C_4	−0.828	0.0	0.0
D_1	−0.526	−0.576	0.078
D_2	0.411	0.360	−0.182
D_3	0.051	0.089	0.123
D_4	0.044	0.0	0.0
c_e (GeV^2)	2.25	2.25	2.56
c_i (GeV)	0.0	0.0	0.0
α (GeV)	−0.8	−0.8	−0.8

waves. In the case of $^1D_2$, all the best fits and most of the second-best fits show the pole lying on sheet III close to and slightly to the left of the $N\Delta$ branch point. This close proximity of the $^1D_2$ pole to the $N\Delta$ branch point on sheet III and its highly inelastic nature raise the possibility of the pole being a bound state of the $N\Delta$ system in the same usual sense as the deuteron in the $^3S_1$ partial waves. In this paper, we subject the $^1D_2$ pole observed within the M-matrix formalism to a simple test[6] to determine whether it can be of the bound-state type as mentioned above. We also, for completeness, report results for the observed $^3F_3$ pole which, unlike the $^1D_2$ pole, lies on sheet IV and to the right of the $N\Delta$ branch point. Its location, one may note, clearly rules out consideration of it being an "$N\Delta$" bound state.

Assuming for the moment that Δ is stable, the right-hand cut structure in the E plane is as shown in Fig. 2. Furthermore, if the NN channel is decoupled from the $N\Delta$, a bound-state pole of $N\Delta$ system in analogy with the deuteron case will lie below the $N\Delta$ square-root branch point. A question which arises naturally is how the position of this pole shifts in the complex E plane when the channel $N\Delta$ is coupled to the lower-lying NN channel. To answer it,[6] one first notes that when A is an M matrix in Eq. (1), the poles of the T matrix correspond to the zeros of the determinant of $M-i\rho$ or, equivalently, to

$$\rho_2=-i\,M_{22}+i\,M_{12}^2/(M_{11}-i\rho_1)\ . \qquad (3)$$

If the channel-coupling interaction M_{12} is set to zero, the presence of a bound state in the $N\Delta$ channel requires M_{22} to be negative. If M_{12} is then assigned a value, establishing a coupling to the NN channel, a pole of this bound-state type will move into sheet II (see Fig. 2). Equivalently, an observed pole on sheet II, approaching the real energy axis below the inelastic threshold as its coupling to the elastic channel is reduced, may be regarded as a bound state in the inelastic channel.[6]

In the coupled-channel system that we are concerned with, the inelastic channel is a quasi-two-body channel. When proper width Γ is assigned to the Δ isobar as in Eq. (2b), a three-body cut appears at $E=M_N+M_N+M_\pi$. The $N\Delta$ branch point corresponding to a mass of $M_0-i\Gamma/2$ for the Δ moves down into the complex E plane (see Fig. 1). A bound-state-type pole, if it exists, would now be

TABLE II. Parameters for the observed $I=1$ $^1D_2$ and $^3F_3$ poles in the M-matrix fits. $|R|$ is the magnitude of the elastic residue of the poles $E_p=E_R-i\Gamma_R/2$. For sheet number, refer to Fig. 1.

Solution type	Pole position E_R (GeV)	Pole position $\Gamma_R/2$ (GeV)	Sheet No.	$\lvert R\rvert/(\Gamma_R/2)$
$^1D_2$ (best)	2.12–2.15	0.08–0.10	III	0.1–0.3
$^1D_2$ (second best)	2.14–2.15	0.05–0.07	III, IV	0.1–0.2
$^3F_3$	2.21–2.22	0.06–0.08	IV	0.1–0.2

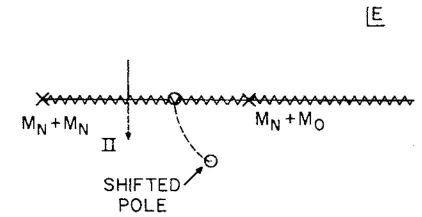

FIG. 2. Right-hand cuts for the NN and $N\Delta$ channels, assuming Δ is stable, and the shift of the $N\Delta$ bound-state pole into sheet II when the coupling with the NN channel is established. The particular trajectory shown is for M_{11} being negative.

present in the vicinity of the $N\Delta$ branch point on sheet III of Fig. 1. The matrix element T_{11} from Eq. (1) is

$$T_{11}=(M_{22}-i\rho_2)/[(M_{22}-i\rho_2)(M_{11}-i\rho_1)-M_{12}^2] \ . \tag{4}$$

Figure 3 shows the position of the observed ${}^1D_2$ pole in the best and the second-best M-matrix fits (see Table II), and its subsequent motion as M_{12} is varied. Table I shows that in each of the ${}^1D_2$ fits, the matrix element M_{12}, although given energy dependence, is essentially constant in the center-of-mass energy region of interest, namely, $1.9 < E < 2.25$ GeV. Consequently, in Fig. 3, we chose to show the pole trajectories by varying the parameter C_1, except near the $M_{12}=0$ end of the trajectory where the other parameters C_m ($m \neq 1$) are simultaneously reduced to reach the $M_{12}=0$ limit. The main point to note here is that, regardless of the particular procedure employed to reduce M_{12} to zero, the pole has a unique position

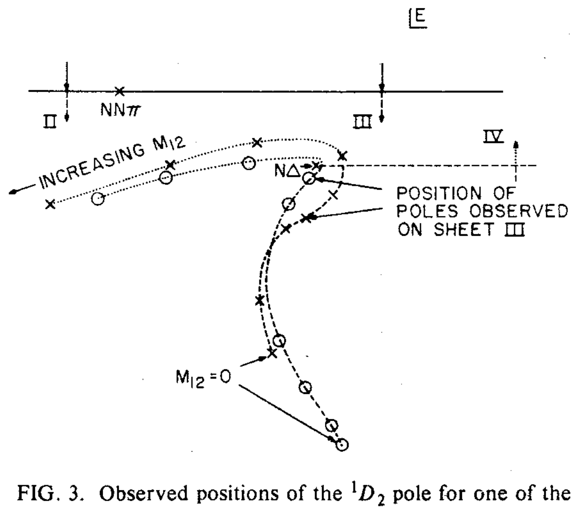

FIG. 3. Observed positions of the ${}^1D_2$ pole for one of the best and one of the second-best fits (see Table I). Trajectories of the pole (crosses for the best and circles for the second best) due to variation in M_{12} are also shown. See text for the method of varying M_{12}.

when $M_{12}=0$. In Fig. 3, the pole's position for $M_{12}=0$ is at a point faraway from the $N\Delta$ branch point and the real-energy axis. The observation that the residue of the T_{11} matrix element at the pole vanishes as $M_{12}\rightarrow 0$ establishes its origin in the part $M_{22}-i\rho_2$ of the denominator in T_{11}. Occurrence of this pole at a complex position in the above limiting case is due to the energy dependence given to M_{22} in the fits. In each of the fits obtained, M_{22} has a negative value below the inelastic three-body threshold. Thus another solution of $M_{22}-i\rho_2=0$ would correspond to a zero on the real energy axis. But in each of these cases a pole corresponding to this zero lies far below the inelastic threshold. Sometimes, it is found to lie even below the elastic threshold. This pole is possibly a spurious pole and anyway not of much significance here since it does not show up in the fit as an influential pole present near the physical energy region. On the other hand, the aforementioned observed pole with a significant influence on the real-energy axis originates from the complex point far away from the physical region. As the channel-coupling interaction M_{12} is established, it approaches the real-energy axis (see Fig. 3) and goes round the $N\Delta$ branch point into sheet IV as the value of M_{12} is increased. In short, the crucial observation that the pole recedes away from, rather than approaches, the real-energy axis as the channel-coupling interaction to the NN channel is reduced seems to cast doubt on the possible interpretation of this observed ${}^1D_2$ pole as a bound-state pole of the $N\Delta$ pole system. One possibility is that the pole is a channel-coupling pole[6] since it originates as a distant singularity in the one-channel case ($M_{12}=0$) and moves close to the physical energy region as the channel-coupling interaction between the two channels represented by M_{12} increases. It is perhaps worthwhile to mention here that, although the pole position changes with the phase-space parameters c_e, c_i, α (being sensitive especially to the parameter c_e which we are able to fix at a value anywhere from 2.0 to 3.0 GeV^2 without altering significantly the quality of the fit), the qualitative nature of the pole's trajectory obtained from varying M_{12} remains the same.

Figure 4 shows the trajectory of an observed ${}^3F_3$ pole in an M-matrix fit obtained by varying the parameters comprising M_{12} (see Table I) in a manner analogous to the ${}^1D_2$ case. In sharp contrast to the ${}^1D_2$ case, this pole moves closer to the real-energy axis as M_{12} is reduced, and is also found to originate in the part $M_{11}-i\rho_1$ of the denominator in Eq. (4). It initially lies on sheet IV and also to the right of the $N\Delta$ branch point. In addition, when $M_{12}\rightarrow 0$, the magnitude of its residue approaches a value very nearly equal to the magnitude of the imaginary part of the pole position, a characteristic reminiscent of a resonance pole in the Breit-Wigner representation of an elastic amplitude. It is therefore quite likely a

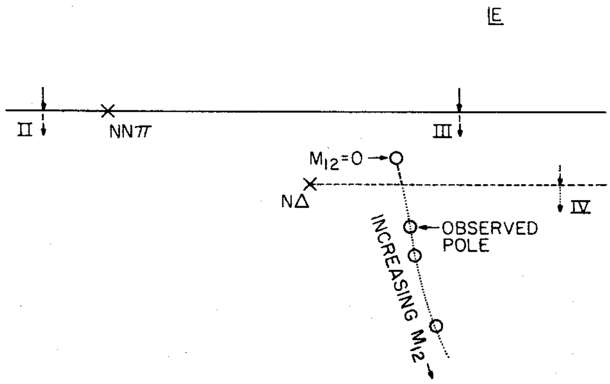

FIG. 4. Observed position of the $^3F_3$ pole in a fit and its subsequent movement as M_{12} is varied. See text for the method of varying M_{12}.

conventional Breit-Wigner resonance pole with a stronger coupling to the $N\Delta$ channel. It is interesting to mention here that authors of Ref. 4, using a separable potential model, also find a resonance pole which, however, in sharp contrast to our case, moves to the left away from the physical energy region, reaching a fixed left-hand singularity as the coupling between the NN and the $N\Delta$ channel vanishes.

In summary, the results of the application of the test to the observed $^1D_2$ and $^3F_3$ poles in M-matrix fits to corresponding phase shifts, while apparently consistent with the idea of the $^3F_3$ pole being a resonance pole, do not appear to support the possibility that the $^1D_2$ pole might be a bound state of the $N\Delta$ system. A more dynamical approach describing the effect of the left-hand cuts in a detailed way is perhaps needed to establish their nature, especially in the $^1D_2$ case. For example, it is well known that the one-pion-exchange effect in the elastic NN channel gives rise to a left-hand cut ~ 5 MeV below the NN threshold. In the elastic $N\Delta$ channel and the inelastic $NN \rightarrow N\Delta$ channel, it also generates branch-point singularities at complex positions in the energy plane, close to the $N\Delta$ branch point in the $N\Delta$ case.[7] In what manner the M-matrix elements of our model incorporate the effects of these singularities is unclear. Explicit consideration of at least those dynamical singularities which are close to the $N\Delta$ branch point appears to be warranted since the observed poles are highly inelastic and close to the $N\Delta$ branch point. One may also want to note that we have not attempted to discuss here the possibility of the observed structure in the $^1D_2$ and the $^3F_3$ partial waves as manifestation of six-quark states. Possible existence of such states is being pursued seriously and is discussed elsewhere.[8]

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