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Specific absorption of a tiny absorbing particle embedded within a nonabsorbing particle

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In a recent paper,[1] analytic expressions for specific absorption (absorption cross section per unit volume of the absorbing material) were derived for the case of a tiny absorbing spherical particle at the center of a nonabsorbing spherical particle. The basis of the derivation was the model of a single-layered sphere (core + shell). The purpose of this Letter is to extend the above results to the case of the tiny absorbing particle located anywhere within the nonabsorbing particle.

Figure 1 shows a single-layered sphere of radii r_1 and r_2 exposed to an electromagnetic plane wave polarized in the x direction. For a nonabsorbing shell (m_2 real), and in the conditions $|m_1|x_1 \ll 1$ and $m_2x_1 \ll 1$, the absorption cross section per unit volume of the absorbing core is given by[1]

$$\sigma_{\text{abs}}/V = \frac{-6\pi m_2^4}{\lambda} \operatorname{Im}\left(\frac{m_1^2 - m_2^2}{m_1^2 + 2m_2^2}\right) \Big/ |D_1^{(h)}|^2, \tag{1a}$$

$$D_1^{(h)} = \zeta_1(x_2)\psi_1'(m_2x_2) - m_2\zeta_1'(x_2)\psi_1(m_2x_2), \tag{1b}$$

where ζ_1 and ψ_1 are Ricatti-Bessel functions of order 1. x_1 and x_2 are size parameters equal to $2\pi r_i/\lambda$, $i = 1,2$. This expression was derived assuming the core to be a perturbation in the scattering of light by the larger homogeneous sphere with parameters m_2,x_2. The effect of the core is to Rayleigh scatter the electromagnetic field at the center of the homogeneous sphere. To see the physics embedded within Eqs. (1a) and (1b), we cast Eq. (1a) in the following form:

$$\sigma_{\text{abs}}/V = -\frac{2\pi}{\lambda} \operatorname{Im}(m_1^2)|F|^2|\mathbf{E}^{(h)}(\mathbf{r} = 0)|^2, \tag{2a}$$

where

$$F = 3m_2^2/(2m_2^2 + m_1^2), \tag{2b}$$

$$\mathbf{E}^{(h)}(\mathbf{r} = 0) = im_2/D_1^{(h)} \exp(i\omega t)\hat{x}. \tag{2c}$$

$\mathbf{E}^{(h)}(\mathbf{r} = 0)$ denotes the electric field at the center of the homogeneous sphere (m_2,x_2). Equation (2c) follows from the general analytic expressions[2] for the internal electric field of a homogeneous sphere. Since the core is tiny, this electric field acts uniformly over it, as a result of which the field inside the core is modified by the factor F. Consequently, $|F|$ times $|\mathbf{E}^{(h)}(\mathbf{r} = 0)|$ represents the magnitude of the electric field inside the absorbing tiny core. This interpretation follows also from a comparison of Eq. (2a) with the following prescription for obtaining an absorption cross section:

$$\sigma_{\text{abs}} = -\frac{2\pi}{\lambda} \int dV \operatorname{Im}(m^2)|\mathbf{E}(\mathbf{r})|^2. \tag{3}$$

$\mathbf{E}(\mathbf{r})$ is the electric field within an inhomogeneous body described by a varying index of refraction m. The integral is carried over the entire volume of the body. This form of σ_{abs} is easily derived from consideration of the Poynting vector, and the energy flows into and out of the scattering body.[6] When Eq. (3) is applied to the single-layered sphere with a tiny absorbing core, Eq. (2a) obtains, provided the magni-

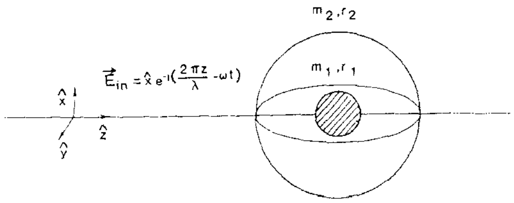

Fig. 1. Single-layered sphere with a tiny core. The parameters of the core and the shell are m_1,r_1 and m_2,r_2, respectively. The equatorial plane and the incident electric field are also shown.

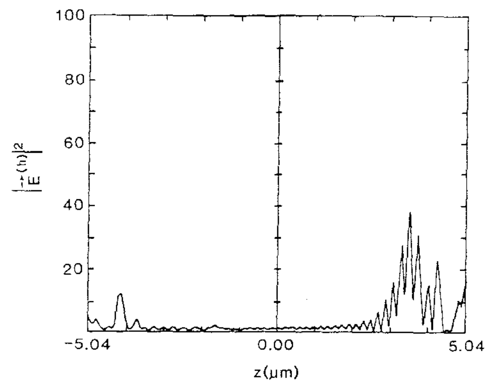

Fig. 2. Plot of $|\mathbf{E}^{(h)}|^2$ along the diameter of the sphere aligned with the z axis. $\mathbf{E}^{(h)}$ is the electric field without the core (m_1,r_1) of Fig. 1. $r_2 = 5.04$ μm.

tude of the field within the absorbing core, $\mathbf{E}(\mathbf{r} = 0)$, is given by

$$|\mathbf{E}(\mathbf{r} = 0)| = |F||\mathbf{E}^{(h)}(\mathbf{r} = 0)|. \tag{4}$$

We now state that this expression for the electric field within a tiny particle in terms of the original electric field provides a natural basis for extending the calculation of the electric field within a tiny particle (and hence the absorption cross section) when the particle is located anywhere within a nonabsorbing dielectric sphere. In other words,

$$\sigma_{\text{abs}}/V = -\frac{2\pi}{\lambda}\,\text{Im}(m_1^2)|\mathbf{E}(\mathbf{r})|^2, \tag{5a}$$

$$|\mathbf{E}(\mathbf{r})| = |F||\mathbf{E}^{(h)}(\mathbf{r})|, \tag{5b}$$

where $\mathbf{r}$ is the position vector of the tiny particle, and $\mathbf{E}^{(h)}(\mathbf{r})$ is the electric field at the position of the particle. The tiny particle, whose size is governed by $m_2 x_1 \ll 1$ and $|m_1|x_1 \ll 1$, behaves like a Rayleigh absorber in an incident field, which is the internal field $\mathbf{E}^{(h)}(\mathbf{r})$. If m_2 is reduced to 1, the larger sphere merges with the outer medium (the vacuum), and $|\mathbf{E}^{(h)}(r)|$ reduces to 1, the magnitude of the incident electric field (see Fig. 1).

Specific absorption, as given by Eqs. (5a) and (5b), is directly proportional to $|\mathbf{E}^{(h)}(\mathbf{r})|^2$, and for a tiny (spherical) soot particle ($m_1 = 2.0\text{–}i0.66$) existing within a spherical water droplet ($m_2 = 1.33\text{–}i0.0$) and an incident wavelength $\lambda = 0.5$ μm,

$$\sigma_{\text{abs}}/V = (16.3)|\mathbf{E}^{(h)}(\mathbf{r})|^2\ \text{m}^2/\text{cm}^3. \tag{6}$$

Figure 2 shows a plot of $|E^{(h)}|^2$ along the diameter of a water droplet of 5.04-μm radius, the diameter taken along the direction of propagation (see Fig. 1). Except for the peaks due to the internal interference effects, the electric field intensity is fairly uniform in the central portion of the plot. The spacing between the peaks is $\sim\lambda/(2m_2)$. In the forward direction near the surface of the water droplet, the peaks are more pronounced due to the focusing effect. The electric field intensity is seen to become as large as 40. If we take a value of ~15 for the average electric field intensity in this focusing region, we see from Eq. (6), that the average value of specific absorption for a soot particle located in this region would be around 250 m^2/cm^3, which is ~10 times as large as the average value of specific absorption (~22 m^2/cm^3) when the soot particle is at the center[7] of the water droplet. We also note here that the maximum possible value of specific absorption for a soot particle in air is only ~13 m^2/cm^3. Figure 3 is a plot for a water droplet of 4.9569-m radius. The electric field intensity near the surface of the water droplet is seen to be dramatically enhanced due to a second-order resonance in the partial-wave mode b_{70}. The width[8] of this resonance is ~4 Å. There is experimental evidence[9] for the existence of such narrow resonances, and, clearly, at such resonant conditions a soot particle clinging to the surface of a water droplet near the forward or backward direction will absorb a lot of energy with a numerical value given by Eq. (6).

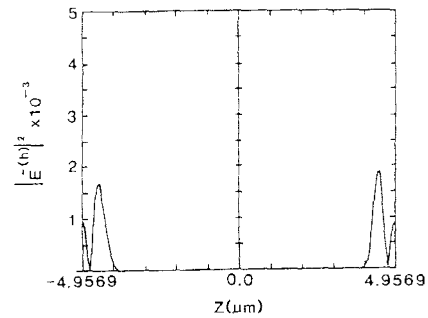

Fig. 3. Same as in Fig. 2 but for $r_2 = 4.9569$ μm.

Summarizing: The primary purpose of this Letter has been to point out a prescription for obtaining numerical values of specific absorption for a tiny (Rayleigh) particle embedded anywhere within another particle. Although physical reasons have been given for why absorption by a small absorbing particle is larger when it is in a larger transparent sphere than when it is in air,[10] no precise prescription for obtaining numerical values seems to have been given. The prescription we provide is in fact general enough to be applicable to any particle (absorbing or nonabsorbing) of any shape within which the tiny absorbing particle resides. In addition, the tiny absorbing particle itself need not be spherical in shape. When it is not spherical, the factor F in Eq. (5b) will have to be modified. The form of F for such shapes as spheroidal and cylindrical can be found in Refs. 3–5. In this Letter, we have illustrated this prescription by applying it to the case of a spherical soot particle embedded within a spherical water droplet. This case is of practical interest in cloud or fog physics.

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7. For a detailed analysis of this case, see Ref. 1; for results based on geometrical optics, see Ref. 5, p. 446.
8. The width of the resonance here is defined to be the shortest range Δr in the radius of the sphere over which the real part of the partial-wave amplitude changes from 0.5 to 1 and then to 0.5 again.
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10. C. F. Bohren, Pennsylvania State U.; private communication (1985), see Ref. 5, p. 446.