spectrum becomes badly nonlinear. This would indicate that the discriminator is probably triggering due to meaningless noise from the photomultiplier tube at these low settings.

## IV. SUMMARY

This experiment performs well and is simple enough to be used in an undergraduate physics lab. With some care given to least squares methods of data analysis and accurate time calibration, it is capable of producing research quality data.

## ACKNOWLEDGMENTS

The assistance of Mr. Harry Clark in machining the scintillator plastic and the help of several other staff members in the development of this apparatus is sincerely appreciated.

* Equipment provided in part by a National Science Foundation Instructional Scientific Equipment Grant.

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AMERICAN JOURNAL OF PHYSICS VOLUME 38, NUMBER 10 OCTOBER 1970

# Visual Appearance of a Moving Vertical Line

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(Received 2 March 1970; revision received 19 May 1970)

A vertical line moving with velocity $v$, when seen, assumes the shape of a hyperbola or parabola accordingly as $v<c$ or $v=c$. At $v>c$, which actually is not possible, the line takes the form of an ellipse, or parts of an ellipse, or even becomes imaginary, depending upon the length of the line, the magnitude of the velocity, and the distance of the viewer from the line. If, however, a line moving with $v<c$ recedes from the viewer, it seems less curved until at infinity it straightens up to look vertical again.

In this paper, I have dealt in detail with the appearance of a moving vertical line and the subsequent changes that take place in it with the passage of time.

Consider a line coincident with the $z$ axis of a frame moving with velocity $v$ as shown in Fig. 1. The light ray I from end 0 reaches the viewer's eye fixed in the laboratory frame at $(0, y, 0)$ after a time interval $y/c$. The ray II from $A$, on the other hand arrives at $P$ at the same time as I after being emitted at an earlier instant $\Delta t$. From the figure, it is clear that

$$c\Delta t+y=[(v\Delta t)^2+y^2+Z^2]^{1/2}. \quad (1)$$

Multiplying both sides by $v/c$ and squaring the resulting expression, we arrive at the equation

$$X^2(1-\beta^2)+2X\beta y-\beta^2Z^2=0, \quad (2)$$

where $X=v\Delta t$. This can be further put in the form

$$[(X+\beta\gamma^2y)^2/(\beta\gamma^2y)^2]-[Z^2/(\gamma y)^2]=1. \quad (3)$$

Equation (3) is in fact the equation of the locus of all points lying on the line, the light rays from which, emitted at different instants ($\Delta t$), arrive at $P$ simultaneously. It is similar to the standard equation[1] of a hyperbola in the $xz$ plane, drawn with the origin shifted to $(x^1, y^1)$. To the viewer, therefore, the line looks like a hyperbola with focus $F\equiv[y/(1+\beta), 0]$ and directrix $D$ given by

$$x=-\beta y/(1+\beta); \quad (4)$$

$y$ here is a constant. The eccentricity $e$ is a function of $v$ and is equal to $1/\beta$.

## DEPENDENCE OF APPEARANCE ON $v$

At $v=0$, $e=\infty$. $F$ tends to $(y, 0)$ and the directrix coincides with the $z$ axis. The hyperbola, consequently, straightens up coinciding with the $z$ axis. However, when the line moves with a finite velocity $v<c$, it assumes the shape of a hyperbola and at $v=c$ transforms into a parabola given by

$$Z^2=2yX. \quad (5)$$

It is interesting to note that with further increase

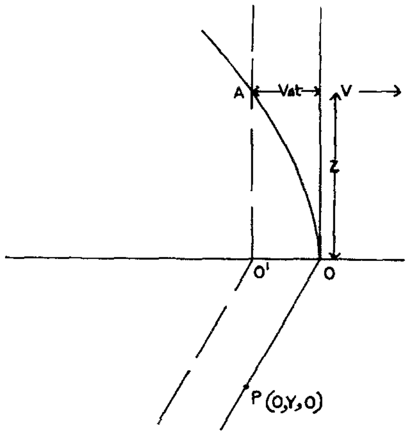


FIG. 1. The appearance of a moving vertical line at $v<c$.

in $v$, which however is incompatible with the theory of special relativity, the curve flattens more, stoops downwards, and eventually changes into an ellipse[2] (Fig. 2) terminating on the $x$ axis at $(2L^2\beta y,\ 0)$. A vertical line of length $Z=Ly$ is represented by the ellipse as shown in the figure, whereas a line of length $Z<Ly$, e.g., $OA$ is seen at two places as two segments $OA'$ and $CB'$ of the ellipse. But when $Z$ exceeds $Ly$, the part $(Z-Ly)$ is imaginary.

## VARIATION IN SHAPE WITH TIME

Investigating into the changes that occur in the appearance of a moving vertical line with the change in the relative location of the viewer, we find that as the line recedes from him, it seems less curved and finally at infinity (if, of course, it is possible to see that far!) it straightens up. This is due to the fact that at large distances, $\Delta t$

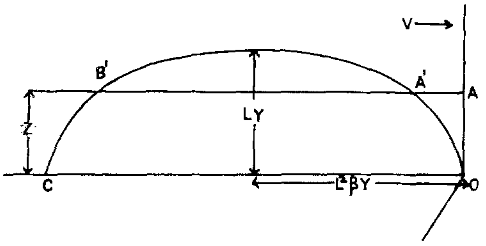


FIG. 2. The appearance of a moving vertical line at $v>c$.

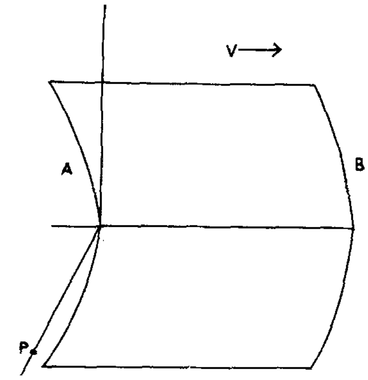


FIG. 3. The appearance of a moving square.

is small. If the viewer is at $(0,\ y,\ 0)$ at time $t=0$, then at time $t=t$, his location with respect to the foot 0 of the line taken as the origin becomes $(vt,\ y,\ 0)$. The focus $F$ of the hyperbola changes to $\{\gamma^2[(v^2t^2+y^2)^{1/2}+\beta vt](1-\beta),\ 0\}$ and the corresponding directrix $D$ to $x=-\beta\gamma^2[(v^2t^2+y^2)^{1/2}+\beta vt](1-\beta)$. The eccentricity still equals $1/\beta$.

The foregoing expressions for $F$ and $D$ show that both of them recede from the origin with the passage of time, with the result that the hyperbola looks less curved. Ultimately, at $t=\infty$, the vertical line is seen to retain its form.

So far we discussed the appearance of a moving vertical line. However, proceeding on the same lines, we could also have derived an expression for the apparent length of a horizontal line. It could also be shown that this may be greater or smaller than its rest length, depending upon the location of the viewer. Figure 3 represents the appearance of a moving square. Note that edge $A$ is more curved than $B$, despite the equality in their eccentricities.

[1] The aforesaid equation of the hyperbola is

$$[(X-x^1)^2/a^2]-[(Z-z^1)^2/b^2]=1.$$

Focus $F$ (lying on the positive $x$ axis) $\equiv[(ae+x^1),\ y^1]$, and equation of corresponding directrix $\equiv(x-x^1)-a/e=0$. $e$ is the eccentricity and equals $(a^2+b^2)^{1/2}/a$.

[2] The equation of the ellipse $\equiv[(X-\beta L^2y)^2/(\beta L^2y)^2]+[Z^2/(Ly)^2]=1$, where $L^2=1/(\beta^2-1)$.