The ωn Cusp in π^-p Elastic Differential Cross-Section.

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Summary. – The effect of the opening of the ωn channel on the π^-p elastic differential cross-section is investigated. It is shown that, despite the ωn's large production rate, the 10 MeV width of ω is large enough to smear the cusp and render it unobservable as a dramatic effect in the π^-p elastic differential cross-section. This is in sharp contrast to a prominent cusp observed at the ηn threshold.

Sometime ago, an experimental group at the Rutherford High Energy Laboratory (1) reported measurements of the differential cross-section for the reactions $\pi^-p \to \pi^-p$, π^0n and ηn in the near-backward direction. An interesting feature of the data was the presence of a sharp cusp at the ηn threshold (1489 MeV of center-of-mass energy) in the elastic reaction $\pi^-p \to \pi^-p$. The cusp was attributed to the strong opening of the ηn channel in a S-wave (1-3). At a higher center-of-mass energy (~ 1723 MeV), the ωn inelastic channel also opens up in a S-wave, and with a fairly large production rate (2,4). Consequently, it is of interest to examine the possibility of a similar cusp-like behavior at the ωn threshold. This letter presents the results of such an investigation.

The π^-p elastic amplitude can be expressed in terms of the well-known no-spin-flip and spin-flip amplitudes, f and g, as

$$M(\theta) = f(\theta) - ig(\theta)\boldsymbol{\sigma}\cdot\boldsymbol{n}\,, \tag{1a}$$

$$\boldsymbol{n} = \frac{\boldsymbol{k}'\times\boldsymbol{k}}{|\boldsymbol{k}'\times\boldsymbol{k}|} = \sin\varphi\boldsymbol{i} - \cos\varphi\boldsymbol{j}\,, \tag{1b}$$

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where $\boldsymbol{k}$ and $\boldsymbol{k}'$ are the initial momentum (along the positive z-axis) and the final momentum (in the $\theta\varphi$ direction), respectively, of the pion in the center-of-mass system. The elastic π^-p differential cross-section is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\frac{\operatorname{Tr}(M^+M)}{2}=|f|^2+|g|^2\,. \tag{2}$$

In terms of the $\pi\mathcal{N}$ elastic partial-wave amplitudes,

$$\begin{cases} f(\theta,k)=\sum\limits_{i=\frac{1}{2}}^{\frac{3}{2}} C_i\{(l+1)a^{(i)}_{l,l+\frac{1}{2}}(k)+la^{(i)}_{l,l-\frac{1}{2}}(k)\}P^0_l(\cos\theta)\,,\\ g(\theta,k)=\sum\limits_{i=\frac{1}{2}}^{\frac{3}{2}} C_i\{a^{(i)}_{l,l+\frac{1}{2}}(k)-a^{(i)}_{l,l-\frac{1}{2}}(k)\}P^1_l(\cos\theta)\,,\end{cases} \tag{3}$$

where i denotes the isospin state of the $\pi\mathcal{N}$ state and the isospin coefficients $C_{\frac{1}{2}}$ and $C_{\frac{3}{2}}$ are, respectively, $\frac{2}{3}$ and $\frac{1}{3}$. P^m_l are the associated Legendre polynomials.

In order to obtain an expression for $M(\theta)$ in the vicinity of the ωn threshold, one first notes that, in general, when an inelastic channel opens up, the elastic partial-wave amplitude (denoted by t below) behaves in the vicinity of the inelastic threshold as [3]

$$t=t_0+i\varrho t'^2+\text{higher orders in }\varrho\,, \tag{4}$$

where t_0 is the value at the inelastic threshold, ϱ is the phase-space factor for the inelastic channel and t' the corresponding inelastic amplitude at the inelastic threshold. In the case of the ωn channel with which we are concerned, there are two possible initial $\pi\mathcal{N}$ states, S_{11} and D_{13} (corresponding amplitudes: $a^{(\frac{1}{2})}_{0,\frac{1}{2}}$ and $a^{(\frac{1}{2})}_{2,\frac{3}{2}}$) which contribute to ωn production in S-wave. Denoting the corresponding inelastic amplitudes by s and d, the total elastic amplitude in the neighborhood of the ωn channel, to first order in ϱ, is

$$M(\theta)=M_0(\theta)+i\tfrac{2}{3}\varrho\big[s^2+d^2\{(3\cos^2\theta-1)+i\sin\theta\cos\theta\boldsymbol{\sigma}\cdot\boldsymbol{n}\}\big]\,. \tag{5}$$

Since ω is unstable (decays into 3π, width $\Gamma=10$ MeV), ϱ is a quasi-two-body phase-space factor which, for our purposes, can be defined by

$$\varrho(E)=\frac{C\Gamma}{2\pi}\int\limits_{m_T=3m_\pi}^{\infty}\frac{\sqrt{E-(m+m_n)}\,\mathrm{d}m}{(m_\omega-m)^2+\Gamma^2/4}\,, \tag{6a}$$

$$C=\left(\frac{2m_\omega m_n}{m_\omega+m_n}\right)^{\frac{1}{2}}, \tag{6b}$$

$m_\pi=0.140$ GeV, $m_\omega=0.783$ GeV, $m_n=0.940$ GeV. Clearly, in the limit $\Gamma\to 0$, $\varrho(E)$ reduces to the expected nonrelativistic expression for the center-of-mass momentum. M_0 in eq. (5) corresponds to the center-of-mass energy $3m_\pi+m_n$. But, in what follows, it is convenient to use the «threshold» energy, $E_0=m_\omega+m_n=1.723$ GeV as the reference point. Thus, we rewrite eq. (5) calling M_0 the value at $E=E_0$:

$$M=M_0+i\tfrac{2}{3}(\delta\varrho)\big[s^2+d^2\{(3\cos^2\theta-1)+i\sin\theta\cos\theta\boldsymbol{\sigma}\cdot\boldsymbol{n}\}\big]\,, \tag{7a}$$

where

$$\delta\varrho(E) = \varrho(E) - \varrho(m_\omega + m_n) . \tag{7b}$$

In the calculations pertaining to the elastic differential cross-section, we choose $\theta = \pi$ for the sake of illustration. This corresponds to scattering in the backward direction as in the ηn case ([1]). Substitution of $\theta = \pi$ in the foregoing results gives $M_0 = f_0$ (see eqs. (1a) and (3)) and

$$M = f_0(\pi) + i(\tfrac{2}{3})(\delta\varrho)[s^2 + 2d^2] , \tag{8a}$$

which implies

$$\frac{d\sigma}{d\Omega} = |f_0(\pi)|^2 - \frac{4}{3}[\mathrm{Im}\,(F)\,\mathrm{Re}\,(\delta p) + \mathrm{Re}\,(F)\,\mathrm{Im}\,(\delta\varrho)] , \tag{8b}$$

where

$$F = f_0^*(\pi)(s^2 + 2d^2) . \tag{9}$$

The behavior of $d\sigma/d\Omega$ clearly depends upon the value of F at the ωn threshold. From eq. (9), knowledge of $f_0(\pi)$, s and d determine F. As far as $f_0(\pi)$ is concerned, it can be calculated from the πN partial-wave amplitudes using eq. (3). The result for its magnitude is $\sim 0.5\,(\mathrm{GeV})^{-1}$. Although s and d are individually unknown, a rough limit on the magnitude of $s^2 + 2d^2$ can be placed from an analysis of experimental information on ωn production in ([2,4]). We first note that the total ωn production cross-section is given by

$$\sigma_i = 4\pi\left(\frac{2}{3}\right)\frac{\mathrm{Re}\,(\varrho)}{k}\,(|s|^2 + 2|d|^2) \tag{10a}$$

implying

$$\sigma_i(E) - \sigma_i(E_0) = \frac{4\pi}{k}\left(\frac{2}{3}\right)\mathrm{Re}\,(\delta\varrho)(|s|^2 + 2|d|^2) . \tag{10b}$$

Considerable simplification results if one notes that the lower limit of the integral in eq. (6) can be extended to $-\infty$, without causing any appreciable error in the calculations. This is so because the width of ω is only 10 MeV and, therefore, the main contribution to $\varrho(E)$ comes from values of m in the vicinity of m_ω. The result under this approximation is

$$\delta\varrho/C = (E - E_0 + i\Gamma/2)^{\frac{1}{2}} - (i\Gamma/2)^{\frac{1}{2}} , \tag{11a}$$

which implies

$$\mathrm{Re}\,(\delta\varrho) = C\left[\left\{\frac{\alpha + (\alpha^2 + \beta^2)^{\frac{1}{2}}}{2}\right\}^{\frac{1}{2}} - \left(\frac{\beta}{2}\right)^{\frac{1}{2}}\right] , \tag{11b}$$

$$\mathrm{Im}\,(\delta\varrho) = C\left[\left\{\frac{(\alpha^2 + \beta^2)^{\frac{1}{2}} - \alpha}{2}\right\}^{\frac{1}{2}} - \left(\frac{\beta}{2}\right)^{\frac{1}{2}}\right] , \tag{11c}$$

where $\alpha = E - E_0$, $\beta = \Gamma/2$.

A fit of eq. (10*b*) in conjunction with the result, eq. (11*b*), to the experimental values of $\sigma_i(E)-\sigma_i(E_0)$ in [2] yields

$$\frac{\sigma_i(E)-\sigma_i(E_0)}{\mathrm{Re}\,(\delta\varrho)}=\frac{4\pi}{k}\left(\frac{2}{3}\right)(|s|^2+2|d|^2)\approx 10\ \mathrm{mb/GeV}\,.$$

Using $k(E_0)=0.6$ GeV, one obtains

$$|s|^2+2|d|^2\approx 1.8\ (\mathrm{GeV})^{-2}\,. \tag{12a}$$

This result implies the inequality

$$|s^2+2d^2|\leqslant 1.8\ (\mathrm{GeV})^{-2}\,. \tag{12b}$$

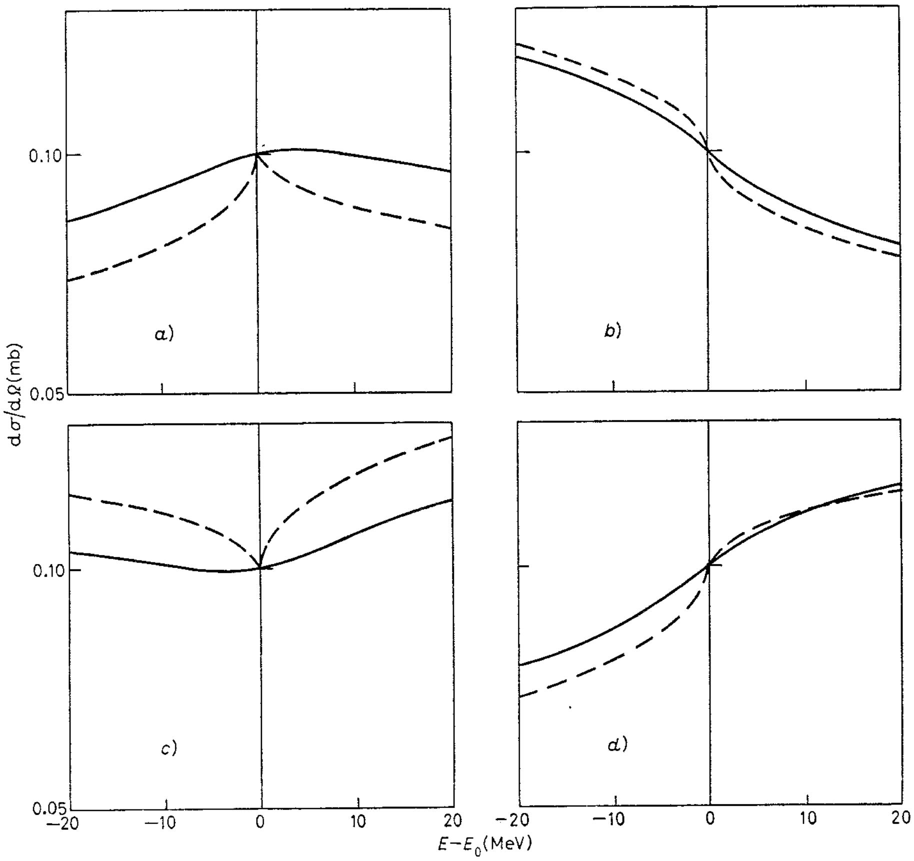

Fig. 1. – The continuous lines are the theoretically calculated behavior of $d\sigma/d\Omega$ (for backward direction in the center of mass) in the vicinity of ωn threshold (E_0) for the phase θ of F equal to *a*) 30°, *b*) 135°, *c*) 240° and *d*) 330°. See text for the definition of F. E is the center-of-mass energy. The broken lines correspond to the behavior in the limit $\Gamma\to 0$, where Γ denotes the width of the ω-meson.

By assuming the upper limit (for the sake of illustration) and using $f_0(\pi) = 0.5$ GeV, the magnitude of F from eq. (9) is

$$|F| = 0.9\ (\text{GeV})^{-3}\,. \tag{13}$$

Because of the dependence of the phase of F on s and d, it is not determined in our analysis. Consequently, in fig. 1, we display the behaviour of $d\sigma/d\Omega$ at the ωn threshold for four different values of the phase (θ) of F. The dashed curve corresponds to the case $\Gamma \to 0$. Comparison of the two curves ($\Gamma = 0$ and $\Gamma = 10$ MeV) for each case of θ shows clearly that, even though the width of ω is only 10 MeV, it is large enough to smear the sharp threshold effect expected in the $\Gamma = 0$ case. Therefore, in an experiment similar to the one performed for the ηn case ([1]), a sharp change in the behavior of the π^-p elastic differential cross-section at or near the ωn threshold may not be visible. Nevertheless, a behaviour of the type depicted in fig. 1 (by continuous lines) is expected, provided the errors on the experimental data points are not too large. That this behavior depends upon the phase θ of F can be explained in the following way: If an extremum exists in the neighborhood of the ωn threshold, as in fig. 1*a*) and 1*c*), its position can be obtained from eq. (8*b*) differentiating $d\sigma/d\Omega$ with respect to α (see eqs. (11*b*) and (11*c*)) and setting the result equal to zero. This set of operations gives

$$\operatorname{tg}\theta = \frac{\operatorname{Im}(F)}{\operatorname{Re}(F)} = \frac{\{(\alpha_m^2 + \beta^2)^{\frac{1}{2}} - \alpha_m\}^{\frac{1}{2}}}{\{(\alpha_m^2 + \beta^2)^{\frac{1}{2}} + \alpha_m\}^{\frac{1}{2}}}\,, \tag{13a}$$

which simplifies to

$$\alpha_m = \beta/\operatorname{tg} 2\theta \tag{13b}$$

and, using $\alpha = E - E_0$, is equivalent to

$$E_m = E_0 + \Gamma/(2\operatorname{tg} 2\theta)\,, \tag{13c}$$

where the quantities with subscript m refer to the values at the extremum. From eq. (13*a*), it is seen that an extremum exists only when $\operatorname{tg}\theta$ is positive, *i.e.* either $0 < \theta < 90°$ or $180° < \theta < 270°$. Furthermore, it is also clear from eq. (13*c*) that the extremum lies to the right if $0 < \theta < 45°$ or $180° < \theta < 225°$ and to the left if $45° < \theta < 90°$ or $225° < \theta < 270°$. The second differential of $d\sigma/d\Omega$ with respect to α gives upon using the result, eq. (13*b*),

$$\frac{d^2}{d\alpha^2}\left(\frac{d\sigma}{d\Omega}\right) = -\frac{8C}{3\sqrt{2}\,\beta^{\frac{3}{2}}}(\operatorname{ctg}\theta)^{\frac{1}{2}}|F|\sin^3\theta\cos^2\theta\,. \tag{14}$$

This result shows that the extremum is a maximum for $0 < \theta < 90°$ and a minimum for $180° < \theta < 270°$. The curves in fig. 1*a*) and *c*) are consistent with the above general result. It is, however, important to mention here that the location of the extremum, as given by eq. (13*c*), is meaningful only if $\operatorname{tg}\theta$ is close to 1, *i.e.* if E_m is close to E_0. This is due to the fact that the expansion in eq. (8*b*) is valid only in the immediate neighborhood of E_0. Finally, one notes that the curves in fig. 1*b*) and *d*) do not display an extremum. Rather, in such cases ($90° < \theta < 180°$ or $270° < \theta < 360°$), a point of inflexion exists near E_0. The rise and fall of $d\sigma/d\Omega$ in fig. 1*b*) and *d*) can be understood by setting its double differential with respect to α to zero and examining subsequently the derivative of $d\sigma/d\Omega$ (with respect to α) at the point of inflexion.

In conclusion, the strong opening of the ωn channel [2,4] gives rise to the possibility of a sharply changing π^-p differential cross-section near the ωn threshold. However, our analysis shows that this expected sharpness (or cusplike behavior) is substantially smeared by the 10 MeV width of ω. Nevertheless, precise experimental measurements of the near backward ($\theta_{c.m.} \simeq 180°$) differential cross-section should be capable of revealing its general behaviour near the ωn threshold, and thus be able to shed some light on the phase of F (defined in the text). Although we have considered only $\theta_{c.m.} = 180°$ (for illustration), experimental differential cross-section data corresponding to different center-of-mass angles can also be analysed within the framework of our model. In fact, a complete analysis involving different center-of-mass angles should in principle be able to yield some useful information on the f and g amplitudes at the ωn threshold. Such information, one notes, can be an important constraint in πN partial-wave analysis.

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