Forbidden decays $\psi' \to \eta + \psi$ and $\psi' \to \pi^0 + \psi$

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It is proposed that the SU(3) violation in the decay $\psi' \to \eta + \psi$ arises from the proximity of the $\bar{D}D$ threshold to the ψ' mass in contrast to the $\bar{F}F$ threshold. In this case the $D^+ - D^0$ mass splitting leads to the SU(2)-violating decay $\psi' \to \pi^0 + \psi$, which is calculated to have a rate not far below its present experimental limit.

Assuming ψ and ψ' are SU(3) singlets[1] and η is an SU(3) octet, the decay $\psi' \to \psi + \eta$ is SU(3)-forbidden. Considering that the decay is forbidden by the Okubo-Zweig-Iizuka (OZI) rule and is a p-wave decay with little phase space, Harari[2] and others have suggested that the observed decay width[3] of about 10 keV cannot be explained by normal SU(3)-violation mechanisms such as η-η' mixing. Harari suggests[4] that η contains a 1% admixture of $\bar{c}c$, allowing $\psi' \to \psi + \eta$ via $\psi' \to \bar{c}c\ \bar{c}c$; however, while this transition is allowed by the OZI rule, it is suppressed by the necessity of producing a $\bar{c}c$ pair from the vacuum. Furthermore, a detailed calculation by Voloshin[5] based on the measured width for $\psi \to \eta\gamma$ gives the result that the $\bar{c}c$ admixture in η makes a negligible contribution to the observed decay width for $\psi' \to \psi + \eta$. In this note we consider an alternative theoretical explanation, namely, that the SU(3) violation can be explained by the small energy gap between ψ' and the $\bar{D}D$ threshold in contrast to the much larger gap to the $\bar{F}F$ threshold. If this explanation is correct, then the splitting between the $D^0\bar{D}^0$ and D^+D^- thresholds leads to an SU(2) violation that would induce the SU(2)-forbidden decay $\psi' \to \psi + \pi^0$. We do not attempt to calculate the absolute rate for $\psi' \to \psi + \eta$, but we do calculate the ratio of the decay rates for $\psi' \to \psi + \pi^0$ to the known rate for $\psi' \to \psi + \eta$; this prediction then can provide a test of our explanation for $\psi' \to \psi + \eta$.

We consider the ψ' state to be given by the standard $c\bar{c}$ state plus an admixture of the continuum states $C_1 = D^0\bar{D}^0$, $C_2 = D^+D^-$, $C_3 = F^+F^-$, assumed calculable by perturbation theory in the following way:

$$|\psi'\rangle = N|c\bar{c}\rangle + \sum_i \int_0^\infty |C_i(E)\rangle \frac{\langle C_i(E)|H'|c\bar{c}\rangle}{E + M_i - M}\, dE\,, \tag{1}$$

where M is the ψ' mass, M_i is the threshold energy for the state i, and N is the normalization factor. The perturbation H' connects $c\bar{c}$ to the continuum $C_i(E)$. For our purposes, states such as $D^{*0}\bar{D}^0$ and $D^{*0}\bar{D}^{*0}$ can be included in C_1 and similarly for C_2 and C_3. As a result of Eq. (1), the matrix element for the transition $\psi' \to \psi + \pi^0/\eta$ may be expressed as

$$\langle P_j\,\psi|T|\psi'\rangle = \sum_i \int_0^\infty dE\, \langle P_j\,\psi|T|C_i(E)\rangle \times \frac{1}{(E + M_i - M)} \langle C_i(E)|H'|c\bar{c}\rangle\,, \tag{2}$$

where P_j is π^0 or η. Further, after the extraction of the Clebsch-Gordan coefficients X_{ji} connecting the states C_i to the octet state $P_j + \psi$, Eq. (2) reduces to

$$\langle P_j\,\psi|T|\psi'\rangle = \sum_i X_{ji} \int_0^\infty \frac{\rho(E)\, dE}{E + M_i - M}\,. \tag{3}$$

The factor $\rho(E)$ is independent of the index i; this corresponds to the crucial assumption in our model that the dynamics is SU(3) invariant, and the large observed SU(3) violation arises from the difference in the threshold energies M_i in Eq. (3).[6] Now with

$$a_i = \int_0^\infty \frac{\rho(E)\, dE}{E + M_i - M} \tag{4}$$

it follows from Eq. (3) that

$$\langle \eta\psi|T|\psi'\rangle = (a_1 + a_2 - 2a_3)/\sqrt{6}\,, \tag{5a}$$

$$\langle \pi^0\psi|T|\psi'\rangle = (a_1 - a_2)/\sqrt{2}\,. \tag{5b}$$

The ratio of the two decays is then given by

$$R = \frac{\Gamma(\psi' \to \pi^0 + \psi)}{\Gamma(\psi' \to \eta + \psi)} = \tfrac{3}{4} p r\,, \tag{6}$$

$$r = (a_1 - a_2)^2 / [\tfrac{1}{2}(a_1 + a_2) - a_3]^2\,, \tag{7}$$

where p is the ratio of phase spaces. Assuming the phase space is given by a standard p-wave form

$$\phi \sim k^3/(1 + k^2a^2)\,,$$

we find $p \approx 5$ if a equals 1 F. In Eq. (4), although

$\rho(E)$ is independent of the index i, its dependence on E is determined by the details of the dynamics. Our goal is to find results which are not very dependent on the dynamics. From Eqs. (7) and (4) we obtain

$$r = \frac{x^2}{(1-\frac{1}{2}x)^2}\,, \tag{8a}$$

$$x = \frac{M_2 - M_1}{M_3 - M_1} F\,, \tag{8b}$$

$$F = \frac{\displaystyle\int_0^\infty \frac{dE\,\rho(E)}{(E+M_1-M)(E+M_2-M)}}{\displaystyle\int_0^\infty \frac{dE\,\rho(E)}{(E+M_1-M)(E+M_3-M)}}\,. \tag{8c}$$

Since $M_1 < M_2 < M_3$, and assuming $\rho(E)$ is positive-definite,[7] it follows from Eq. (8c) that

$$1 < F < (M_3 - M)/(M_2 - M)\,. \tag{9}$$

From Eqs. (6) and (8) we then obtain

$$\frac{3}{4}p\, \frac{\left[\dfrac{M_3-M}{M_2-M}\right]^2 \left[\dfrac{M_2-M_1}{M_3-M_1}\right]^2}{\left(1-\dfrac{1}{2}\,\dfrac{M_2-M_1}{M_3-M_1}\,\dfrac{M_3-M}{M_2-M}\right)^2} > R > \frac{3}{4}p\left[\frac{M_2-M_1}{M_3-M_1}\right]^2. \tag{10}$$

Setting $p=5$, $M(D^0) = 1863$ MeV, $M(F^+) = 2040$ MeV, and $M(D^+) - M(D^0) = 5$ MeV (Ref. 8) we find

$$0.2 > R > 0.003\,. \tag{11}$$

The present experimental limit[3] is $R < 0.04$; thus our model implies that the decay $\psi' \to \psi + \pi^0$ should be found if experiments can be improved by an order of magnitude.

To test the self-consistency of our model we consider other possible final states resulting from the admixed states C_i. We assume that final states, such as N pions, that contain no $\bar{c}c$ pair are suppressed by an OZI rule. For other final states, such as $\psi\pi\pi$, it is necessary to include the SU(3)-invariant piece of the admixed states in addition to the SU(3)-noninvariant piece we have considered so far. The former, which is proportional to $(a_1+a_2+a_3)/\sqrt{3}$, is always larger than the latter, which is proportional to $(a_1+a_2-2a_3)/\sqrt{6}$, since all a_i are positive. In order that the states C_i do not contribute too much to the SU(3)-invariant decays, we impose the requirement that

$$\frac{a_1+a_2+a_3}{\sqrt{3}} \leq 4\,\frac{a_1+a_2-2a_3}{\sqrt{6}}\,. \tag{12}$$

TABLE I. Lower limit on $R(\times 100)$ for various mass combinations.

$M(D^+)-M(D^0)$ (MeV) \ $M(F^+)$ (GeV)	1.975	2.1
4	2	1
5	3	1.5
6	4	2

With this restriction on $\rho(E)$ we obtain for a lower limit on R the values shown in Table I. Such a restriction is also needed so that the SU(3)-invariant transitions from C_i to $\psi + \eta'$ combined with a reasonable amount of η'-η mixing do not seriously modify our original estimate Eq. (5a) for $\psi' \to \eta + \psi$.

Our conclusion is that if the SU(3)-violating decay $\psi' \to \psi + \eta$ is to be explained by the proximity of the $\bar{D}D$ threshold to ψ' then we expect the ratio R to be of the order 1% or larger, not far below the present experimental limit.

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[1]Evidence that ψ is an SU(3) singlet is summarized by G. Feldman, in Proceedings of Summer Institute on Particle Physics, SLAC Report No. 198, 1976 (unpublished). The standard charmonium theory requires that both ψ and ψ' be SU(3) singlets.

[2]H. Harari, Phys. Lett. 60B, 172 (1976).

[3]W. Tannenbaum *et al.*, Phys. Rev. Lett. 36, 402 (1976).

[4]See also C. Rosenzweig, Phys. Rev. D 13, 3080 (1976).

[5]M. B. Voloshin, Report No. ITEP-142 (unpublished).

[6]In order to include $D^*\bar{D} + \bar{D}^*D$ and $D^*\bar{D}^*$ states, we are also assuming $M(D^{*+}) - M(D^{*0}) = M(D^+) - M(D^0)$, which is not inconsistent with present knowledge. Our results depend primarily, however, on $M(D^+) - M(D^0)$.

[7]A detailed dynamical model by E. Eichten and collaborators suggests that $\rho(E)$ may become negative for large values of E. However, our results are insensitive to the behavior of ρ at such large values of E. We are indebted to Dr. Eichten for showing us these results before publication.

[8]Recent results give $M(D^+) - M(D^0) = 5.1 \pm 0.8$ MeV, $M(D^{*+}) - M(D^{*0}) = 2.6 \pm 1.8$ MeV. Within errors a common mass difference between 4 and 5 MeV fits these data. Preliminary evidence on the F^+ suggests a mass of 2.040 ± 0.060 GeV. A range of values consistent with these data is used in Table I.