Internal and near-surface scattered field of a spherical particle at resonant conditions: comments

Ramesh Bhandari

New Mexico State University, Physics Department, Applied Lasers/Optics Group, Las Cruces, New Mexico 88003.

Received 7 April 1986.
0003-6935/86/152464-03$02.00/0.

The paper[1] by Chylek *et al.* appeared recently in *Applied Optics*. The internal electric field of a dielectric sphere is a popular topic which has received a lot of attention in the recent past.[2,3] The purpose of this Letter is to comment on certain features of internal electric field intensity reported in Ref. 1.

We first note that in Ref. 1 light incident on a dielectric sphere is assumed to be unpolarized. If the direction of incident beam is taken to be the z axis, the unpolarized light can be considered to be an incoherent equal mixture of light polarized in the x and y directions. Since the x and y directions for the polarization of incident light yield similar results in the determination of electric field intensity $|\mathbf{E}|^2$, we shall for the purpose of discussion assume that the incident light (of unit amplitude) is polarized in the $\hat{x}$ direction only. With this assumption in mind, we now find that the internal electric field **E** along the diameter of the dielectric sphere parallel to the direction of propagation (the z direction) is given by

$$\mathbf{E} = \frac{\exp(i\omega t)}{2kr} \sum_{n=1}^{\infty} (\mp i)^n (2n+1)[d_n \psi_n(mkr) \pm i c_n \psi_n'(mkr)]\hat{x} \quad (1)$$

according as $\cos\theta = \pm 1$. We are following the notation of van de Hulst.[4] ψ_n and ψ_n' are the familiar Ricatti-Bessel functions. $k = 2\pi/\lambda$, where λ is the wavelength of incident radiation, ω is the angular frequency of radiation, m is the refractive index of the dielectric sphere. c_n and d_n are the internal scattering coefficients which depend sensitively on the refractive index m and the size parameter ($= 2\pi a/\lambda$) of the sphere. a is the radius of the sphere. At sharp resonant conditions, either the c_n or d_n coefficient becomes extremely large, giving rise to huge peaks near $r = a$ in the internal electric field intensity plots along the above diameter.[1,3] In Ref. 1, a resonance in c_n or d_n is denoted by $TM_{n,l}$ or $TE_{n,l}$, respectively, where n is the partial-wave number and l is the order of the resonance within the partial wave. Clearly, from Eq. (1), a resonance in a TE or TM mode will make a significant contribution in terms of huge peaks when $\psi_n(mkr)$ or $\psi_n'(mkr)$ starts to assume its oscillatory character. This occurs when $mkr \sim n$ or, equivalently, when $r/a \sim n/mx$. Chylek *et al.* have given an empirical formula for the position of the first huge peak. The formula is devoid of any explicit dependence on the refractive index m as it is based on one particular case studied by them. Therefore, because of this lack of generality, it is not useful. We now show that analytic expressions giving the locations of the peaks do exist. Confining our attention first to a resonance in the c_n coefficient (or the TM mode) and ignoring the relatively small contribution of the other (nonresonating) terms in the series [Eq. (1)], we see that the first huge peak in the plot of $|\mathbf{E}|^2$ along the z axis corresponds to the first maximum or minimum in the function $\psi_n'(mkr)$. In general, if $r_{M,j}$ denotes the jth extremum of ψ_n', then $\psi_n''(mkr_{M,j}) = 0$ or equivalently from the Riccati-Bessel equation: $\psi_n''(z) + [1 - n(n+1)/z^2]\psi_n(z) = 0, \psi_n(mkr_{M,j})\,[n(n+1)/(mkr_{M,j})^2 - 1] = 0$. The first zero of $\psi_n'(mkr)$, or the position of the first peak, is given by setting the second factor equal to zero or equivalently by

$$f_{M,1} = r_{M,1}/a = \sqrt{n(n+1)}/(mx), \quad (2a)$$

where $x = 2\pi a/\lambda$ is the size parameter of the dielectric sphere. The position of succeeding peaks $r_{M,j}$ when l (the order of the resonance) > 1 would be given by the zeros of $\psi_n(mkr)$, i.e., by

$$f_{M,j} = r_{M,j}/a = \psi_{n,j-1}/(mx), \qquad j = 2,3,\ldots l, \quad (2b)$$

where $\psi_{n,\nu}$ denotes the νth zero of the ψ_n function. Similarly, for a TE resonance, the extrema of $\psi_n(mkr)$ in Eq. (1) define the position of the peaks in the $|E|^2$ plot along the z axis. In other words, if $r_{E,j}$ denotes the position of the jth peak due to a $TE_{n,l}$ resonance,

$$f_{E,j} = r_{E,j}/a = \psi_{n,j}'/(mx), \qquad j = 1,2,\ldots l, \quad (3)$$

where $\psi_{n,j}'$ denotes the value of the jth zero of the derivative of the ψ_n function. Since the extrema of the ψ_n function are the zeros of the ψ_n' function, Eq. (1) also implies that the peaks in the TE mode resonance will coincide with the minima of the TM mode resonance for the same partial wave number n, provided the plot of $|E|^2$ vs r, the radial distance. If, however, the values of x at which the $TE_{n,l}$ and $TM_{n,l}$ resonances occur are very close to each other, the above coincidence would occur in the plot of $|\mathbf{E}|^2$ vs r/a also. This is verified in Figs. 2 and 3 of Ref. 1 where $(TE)_{53,1}$ and $(TM)_{53,1}$ are displayed. The minimum is not exactly zero[5] since the contribution of the nonresonating terms in Eq. (1) are also included in the calculation of $|E|^2$. Nevertheless, these nonresonating terms, which constitute a background, do not affect the positions of the peaks given by the aforementioned equations since the resonance is sharp. This overlapping of the maxima and minima, as discussed above, also explains the fact that a peak in the TM mode occurs at a value of r less than that for the corresponding peak in TE mode (see Figs. 2 and 3 of Ref. 1).

The formulas we have given, Eqs. (2a, 2b), and (3) are general equations valid for arbitrary values of m,n,x, where x is the size parameter corresponding to resonant conditions. Their forms, at first, might suggest that $f_{M,j}$ or $f_{E,j}$ decreases inversely as the magnitude of the refractive index m. But investigations show that the size parameter x corresponding to the first resonance ($l = 1$) is related to the partial wave number n through the relation $n = \alpha m x$, where α is a numerical factor less than 1. For m in the 1.1–1.7 range and partial wave number n in the 10–76 range, we find α varying from ~0.7 to ~0.9. The large values are obtained when x is large and m is small. From Eq. (2a), we clearly see that $f_{M,1} \cong \alpha$, indicating that the huge peaks that appear in the internal field intensity plots at resonant conditions are always confined near the surface. From the foregoing relation connecting n to m,x, we also find that for a given partial wave number n, the size parameter x of a resonance is essentially inversely proportional to the refractive index m. Also we observed

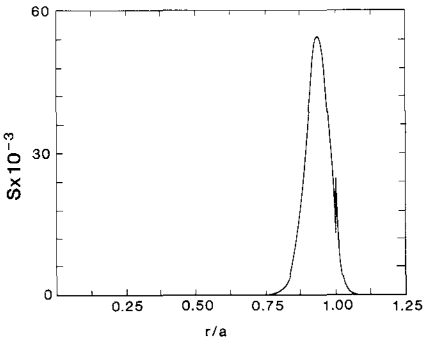

Fig. 1. Source functions S ($= |\mathbf{E}|^2$) averaged over angles for the $TM_{53,1}$ resonance of Ref. 1. r is the radial distance, and a is the radius of the sphere. The amplitude of incident light is unity.

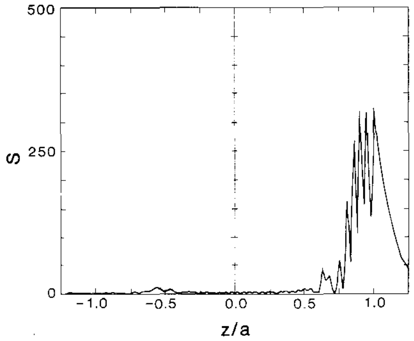

Fig. 2. Source function S for the size parameter $x = 40.7960$ of Ref. 1, but for refractive index, $m = 1.77$. It is calculated for the z axis, the direction of propagation of incident light.

that the resonance became thinner as m increased for a given value of n, implying that the imaginary part of the complex pole becomes smaller as m becomes larger. In fact, as $m \to \infty$, $x \to 0$, and in this limit the poles lie on the real axis. This case is discussed very elegantly by van de Hulst.[4]

It should be emphasized here that the second apparent peak at $r = a$ in the $TM_{53,1}$ mode of Ref. 1 is not strictly a peak because it is not a conventional maximum. The rise after the first minimum is a consequence of the oscillatory behavior of the $\psi_n'(mkr)$ function as explained above. The value of the source function at the interface ($r/a = 1$), both inside and outside the particle, is the same due to the continuity of the electric field, since it is tangential at $r = a$ in the equatorial plane.[6] The second peak is at best a cusp peak. We disagree with the authors when they say: "Each peak . . . corresponds to a spherical shell of high value of the source function." To determine whether a high peak corresponds to a shell of high electric field intensity, it is necessary to calculate $|\mathbf{E}|^2$ as a function of r averaged over angles. In Fig. 1, we show a plot of $|\mathbf{E}|^2_{avg}$ vs r for the $TE_{53,1}$ resonance of Ref. 1. There is only one large peak within the particle, and no minimum and a rise is observed as in the $|\mathbf{E}|^2$ vs z-axis plot. The absence of these features in Fig. 1 is due to the smearing effect of the radial component of the electric field. The presence of the radial component is also manifested in the discontinuity at $r = a$. The intensity just outside is larger because the radial component of the electric field just outside is increased by the factor of m, the refractive index of the sphere. The cusp peaks clearly do not give rise to a separate shell of high intensity. If, however, the discontinuity peak of Fig. 1 is to be regarded as a second shell, it should be emphasized that it is due to the discontinuous nature of the radial component of the electric field and not the cusp peaks. We also point out here that the sharp downward decline in the electric field intensity outside the sphere at resonant conditions, observed in each of Figs. 2–4 of Ref. 1, is mathematically a consequence of the spherical Bessel function of the second kind which is decreasing rapidly at $r = a$ from its infinite value at $r = 0$.

At nonresonant conditions, the presence of a large broad peak outside the particle (due to the focusing effect) in Figs. 2–4 of Ref. 1 is an interesting result, characteristic of particles with low refractive indices only. When the value of the refractive index is low, the focusing effect is not sharp, and there is a large broad peak in the forward direction outside the particle. In addition, one also sees very large wiggles inside the particle which apparently are due to the rays being focused after internal reflection from the back side of the particle (see also Ref. 3). When the refractive index is increased, the broad peak becomes larger and narrower and penetrates the surface $r = a$. Figure 2 shows the partial penetration of the broad peak of Ref. 1 when m is increased to 1.77. The part of the broad peak inside the particle has broken up into wiggles (of corresponding height) due to internal interference effects. As m is increased further, the part of the peak lying outside moves inward breaking up also into wiggles with increasing penetration. Finally, the whole entity (the collection of high wiggles) lying inside, which grows in size and becomes narrower, moves toward the center of the particle with increasing m. In Fig. 2, one also notices a group of smaller peaks to the left of the center of the particle. This set of peaks is always present. Our calculations also show that they move toward the center of the particle as m is increased. This behavior is consistent with the focusing of rays after internal reflection from the back side of the particle.

References

1. P. Chylek, J. D. Pendleton, and R. G. Pinnick, "Internal and Near-Surface Scattered Field of a Spherical Particle at Resonant Conditions," Appl. Opt. **24,** 3940 (1985).
2. See, for example, P. W. Dusel, M. Kerker, and D. D. Cooke, "Distribution of Absorption Centers Within Irradiated Spheres," J. Opt. Soc. Am. **69,** 55 (1979); P. R. Conwell, P. W. Barber, and C. K. Rushforth, "Resonant Spectra of Dielectric Spheres," J. Opt. Soc. Am. **A1,** 62 (1984); J. R. Probert-Jones, "Resonance component of Backscattering by Large Dielectric Spheres," J. Opt. Soc. Am. **A1,** 822 (1984); W. M. Greene, R. E. Spjut, E. Bar-Ziv, A. F. Sarofim, and J. P. Longwell, "Photophoresis of Irradiated Spheres: Absorption Centers," J. Opt. Soc. Am. **B2,** 998 (1985).
3. R. Bhandari, "Specific Absorption of a Tiny Absorbing Particle Embedded Within a Nonabsorbing Particle," submitted to Appl. Opt. **25,** (Month, 1986).
4. H. C. van de Hulst, *Light Scattering by Small Particles* (Dover, New York, 1981).

5. In fact, the minimum following the first large peak in Fig. 3 of Ref. 1 is significantly different from zero because of the insufficient smallness of the step size to reveal the local details.
6. This is easily seen from the analytic expressions for the electric field intensity within the equatorial plane. The equatorial plane is the yz plane in our discussion.